Individual arrangement[1]

The number has not been discovered, it was invented[2], and there's nothing she's equal in the realm of inventions

(Ernst Jünger)

Language is used to transmit messages. The "meaning" of a message should be given from the informant to the recipient of the message. A written, oral or otherwise formulated message is taken on its own not yet "meaningful". It is a "physical reality" such as a handwritten piece of paper, a monitor covered with signs, an acoustic phenomenon such as language etc. It can get a "meaning" only through the informant himself or through a receiving person. Such a " meaning" is therefore dependent on individuals.

Starting from these considerations, one comes to conclusions for the set theory. It turns out that all proofs of the existence of uncountable sets contains a contradiction. As an example the set of real numbers between 0 and 1 is checked. Finally, it follows that every proof of the existence of sets with a cardinality $\aleph > \aleph_0$ contains a contradiction.

Through most of the papers on this website runs like a red thread the idea of relativity of all truth. A written, oral or otherwise transmitted statement can only by a possible person, who reads, hears or receives otherwise this statement, get sense and be considered as "true" or as "not true"[3]. The same applies to terms such as "message" or "information". They also can get sense only by reference to a person receiving those message or information just like the sense of a punch card is only recognized by a punch card reader[4]. Therefore are statements, messages or information alone only "potential" meaningful. They get an "actual" sense only through a person, which receives them.

As soon as one is prepared in principle to follow such train of thought, immediately a limit for meaningful true information M follows. One condition for such meaningful true information is evident, it must at least be possible, that a sufficient small time interval IT = $(T,T+\varepsilon)$ exists, in which some person P is prepared to denote the information M as "true". Obviously $\varepsilon = 0.01$ sec would be sufficient. Considering this, it is easily possible to arrange countable all meaningful true messages M with the help of countable arrangements of all possible Messages M, all possible time intervals IT and all possible persons P. In such an arrangement any designation of M as true

---

[1] The paper is based on "Wolff, 'Zur Problematik der absoluten Überabzählbarkeit', PHILOSOPHIA NATURALIS, Bd. 13, Heft 4, 3. Vierteljahr 1972, S 399 - 404". (Printing error in 6.1, lines 5 and 6)

[2] "The mathematician is an inventor, not a discoverer". Ludwig Wittgenstein, Bemerkungen über die Grundlagen der Mathematik, Werkausgabe Bd. 6, Teil I, 168. Suhrkamp Taschenbuch Wissenschaft 506.

[3] To "sense of a statement," cf. Ludwig Wittgenstein, Werkausgabe vol 6, 'Bemerkungen über die Grundlagen der Mathematik', Suhrkamp Taschenbuch Wissenschaft 506.

[4] cf "L'homme ordinateur", http://www.fam.tuwien.ac.at/~wolff/

applies always only to exactly that person P and this time interval IT. Any designation of M by another person PA for instance by the author or by the reader of this paper is irrelevant.

We start with a countable arrangement AO(M) of all possible Messages M and restrict ourselves - as we shall show later, without loss of generality - for now on written messages. A "message M of size n" shall be a square arrangement consisting of n^2 "elementary squares" of side length 1/100 mm, each of which is either white or black, arranged in n rows of n positions. To a white elementary square we associate the number 1, to a black the number 2. That elementary square standing in row j at the position k we denote by a_{jk}. Any message M of size n is then clearly represented by the decimal number $a(M) = 0,a_{11}a_{12}...a_{1n}a_{21}a_{22}...a_{jk}...a_{nn}$. In addition, we arrange all the messages M of size n in groups according to their size n and then within each group according to the size of a(M) in a countable arrangement AO(M). Such a message M of size n could be for instance a white square piece of paper or a white square screen with the side length n/100 mm covered with black characters (letters, formulas, etc.). Apparently all possible black-and-white information can be represented by such messages.

That the restriction on such black-and-white information leads to no loss of generality, is evident by the following considerations: In general, the term "message" means, that an information is transmitted from one person to another. If one omits the restriction to black-and-white information remains as the task of every message the transmission of the contents of the message to a person so, that he can capture the content of the message with the help of his senses. It is easily seen, that all senses can absorb only a countable number of impressions. The main principle for us here, the countability of all possible information, is maintained even in absence of the above mentioned restriction to optically detectable black-and-white messages.

As a next step, we show, that all possible messages M together with all possible time intervals IT and all possible persons P who call in any such time interval any possible message M "true" can be arranged in a countable arrangement AO(M,IT,P). The triple (M,IT,P) here means, that the person P, if he reads the Message M in the time interval IT in fact, he will call M "true".

For this purpose we choose a coordinate system in the space-time-universe (three spatial coordinates, one time coordinate) and divide it into space-time elements STE. A space-time element STE is a (four dimensional) unit-cube UC of side length 0,01 mm (three spatial coordinates) and the duration of 0,01 sec. (one time coordinate). With the help of the previously chosen coordinate system in the space-time-universe all unit-cubes UC can be arranged in a countable arrangement AO(UC).

We can assume that any person who in any possible time interval IT denotes any possible message M as true occupies there a certain volume in the space-time-universe. The size of a unit-cube UC was chosen small enough, so that certainly at least one unit-cube is located entirely in that volume. By this unit-cube UC the triple (M,IT,P) - which means, that P, if he reads M in IT in fact, he denotes M as "true" - is therefore clearly defined. These considerations apply to all possible messages M, all possible time intervals IT and all possible persons P. So all possible messages M will be presented to all possible persons P in all possible time intervals IT for comment.

With the help of the countable arrangement AO(M) of all possible messages M and the countable arrangement AO(UC) of all possible unit-cubes UC as requested a countable order AO(M,IT,P) of all those cases can be found in which any person P denotes as "true" any message

M in any time interval IT. It should be noted that it is spoken only on judgements made by persons P about the "truth" of messages M in a time interval IT. A possible "truth" or "non-truth" of M in another time interval ITO or for other persons PO is not considered. Even the possible "meaning of a message M" for the author and/or the reader of this paper is irrelevant. Hence the term "individual" arrangement.

One example are messages M = M[RN(0,1)], describing a real number RN(0,1) between 0 and 1 clearly and consistently[5]. The triple {M[RN(0,1)],IT,P} states in this case, that in the time interval IT the person P denotes the statement "M[RN(0,1)] describes a real number between 0 and 1 clearly and consistently" as true. From the countable number of all triples (M,IT,P) are filtered out those which state, the message M = M[RN(0,1)] describes a real number between 0 and 1 for the person P in the time interval IT clearly and consistently. This leads to a countable arrangement AO{M[RN(0,1)],IT,P} of all those triples {M[RN(0,1)],IT,P}. Each of this triples describes a real number between 0 and 1 clearly and consistently. From the countable arrangement AO{M[RN(0,1)],IT,P} one obtains simultaneously a countable arrangement AO[RN(0,1)] of real numbers between 0 and 1. We call this arrangement AO[RN(0,1)] "individual arrangement of all real numbers between 0 and 1" and claim, it is complete.

In fact every attempt to show the incompleteness of AO[RN(0,1)] by specification of a real number between 0 and 1 allegedly not contained in AO[RN(0,1) must fail. Indeed , if a critic of the completeness of AO{M[RN(0,1)],IT,P} - we call him PC, critical person, - says in any time interval ITC a real number $r = r_c$ between 0 and 1 - for instance the diagonal number obtained by the second diagonal argument of Cantor applied to the arrangement AO[RN(0,1)] - is not included in this arrangement, that means $r_c \notin$ AO{M[RN(0,1)],IT,P}, then we ask him to bring this claim in the form of a message MC - for instance a written description of the diagonal number obtained by the second diagonal argument of Cantor applied to the arrangement AO[RN(0,1)]. This message MC says in the opinion of the critic, it is true that the real number r_c is between 0 and 1and it is described clearly and consistently by MC. But then from our definition of AO{M[RN(0,1)],IT,P} it follows $r_c \in$ AO{M[RN(0,1),IT,P}. The critic claims, however, $r_c \notin$ AO{M[RN(0,1)],IT,P} and this is a contradiction[6].

In the same way the countability of other allegedly not countable sets can be proved[7].

The countability of all triples {M[RN(0,1)],IT,P} should be understood only as a statement about the cardinality of the set of all such triples, and that is the same as the cardinality of the set of the real numbers between 0 and 1. This statement should not be misunderstood as a possibility of a real complete arrangement of the real numbers[8]. Such an arrangement is in principle not possible because it requires the knowledge of the evaluation of all possible messages M by all possible persons P in all possible time intervals IT. But the knowledge of such evaluations by other

---

[5] cf "Das Münchhausen-Paradoxon" et al., http://www.fam.tuwien.ac.at/~wolff/

[6] The actual core of the considerations.

[7] cf "Ein Widerspruch in überabzählbaren Mengen" et al., http://www.fam.tuwien.ac.at/~wolff/

[8] A very common misconception

persons PO is in principle not accessible to the author and to the reader of this paper. Quite apart from the fact that any possible person is able to read only messages of finite size. Nevertheless such sets can be build completely step by step through every possible person P[9].

A real complete, so "actual" arrangement of real numbers is therefore contrary to the fact that while the set of all triples {M [RZ (0.1)], I, P} is countable, the triples themselves are only "potentially" meaningful. They can thus lead only to an "potential" but not to an "actual" arrangement.

It is therefore not possible to specify a countable arrangement of the real numbers between 0 und 1 actually. But as an essential statement remains the thesis: "Any existence proof of sets with a cardinality $\aleph > \aleph_0$ contains a contradiction".

[9] cf "Ein schrittweiser Aufbau des Kontinuums", http://www.fam.tuwien.ac.at/~wolff/